(61) $g(x) = \frac{x^2}{x^2+1}$ $\quad f(x) = \frac{1}{\sqrt{x^2-x}}$ $\quad$ نوشته ✓ 1

$$fog = f(g(x)) = \frac{1}{\sqrt{g^2-g}} = \frac{1}{\sqrt{\frac{x^4}{(x^2+1)^2} - \frac{x^2}{x^2+1}}} \Rightarrow \sqrt{\frac{1}{\frac{x^4 - x^2(x^2+1)}{(x^2+1)^2}}}$$

$$= \frac{x^2+1}{x\sqrt{-x^2}} \qquad D_{fog} = \phi$$

---

(62) $y = \frac{1}{2}(e^x - e^{-x}) \longrightarrow y' = \frac{1}{2}(e^x + e^{-x})$ $\quad$ نوشته ✓ 1

$$m_{\perp} = -\frac{1}{y'} = \frac{-2}{e^x + e^{-x}} = \frac{-2}{2+\sqrt{3} + \frac{1}{2+\sqrt{3}}} = \frac{-2}{\frac{(2+\sqrt{3})^2+1}{2+\sqrt{3}}}$$

$$= \frac{-2(2+\sqrt{3})}{8+4\sqrt{3}} = \frac{-2(2+\sqrt{3})}{4(\sqrt{3}+2)} = -\frac{1}{2}$$

---

(63) $f(-3) = 2(-27) + 3(9) - 12(-3) = -54 + 27 + 36 = 9$

$f(2) = 2(8) + 3(4) - 24 = 24 = f_{max}$ $\quad$ نوشته ✓ 4

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(64) $y = x^3 - 3x^2 + 3x$ $\quad A(0,1)$ $\quad$ تست 4

$y = (x-1)^3 + 1$ $\quad y = mx + 1$ معادله خط

$mx + 1 = (x-1)^3 + 1 \longrightarrow (x-1)^3 - mx = 0$ مماس

$3(x-1)^2 = m = \frac{y_0 - 1}{x_0 - 0}$

$3(x_0-1)^2 = \frac{(x-1)^3}{x_0} \longrightarrow 3x_0 = x_0 - 1 \longrightarrow 2x_0 = -1 \longrightarrow x_0 = -\frac{1}{2}$

$m = 3\left(-\frac{1}{2} - 1\right)^2 = 3\left(-\frac{3}{2}\right)^2 = \frac{27}{4}$

---

(65) $f(x) = x^{\frac{1}{7}} \longrightarrow f'(x) = \frac{1}{7} x^{-\frac{6}{7}}$ تست 2

$f(x + \Delta x) - f(x_0) = f'(x_0)\,\Delta x$

$x_0 = 2^7 = 128$, $\Delta x = -1.4$

$f(2^7 - 1.4) = 2 + \frac{1}{7}(2^7)^{-\frac{6}{7}} \times (-1.4)$

$\sqrt[7]{126.6} = 2 + \left(\frac{2^{-6}}{1}\right)(-1.4) = 2 - \frac{1.4}{64} \times \frac{1}{7} = 1.99687$

---

(66) $y = \ln\left[-\log_2\left(\log_4 x\right)\right] = \ln\left(-\frac{\ln(\log_4 x)}{\ln 2}\right)$

$y = \frac{1}{\ln 2}\ln\left(-\frac{\ln x}{\ln 4}\right) \longrightarrow y' = \frac{1}{\ln(2)}\left[\frac{-\frac{1}{x\ln 4}}{\frac{\ln x}{\ln 4}}\right]$

$y' = \frac{1}{\ln 2}\left[\frac{-\frac{1}{x\ln 4}}{-\frac{\ln x}{\ln 4}}\right] = \frac{1}{2(\ln 2)^2}$ سه نمره 3

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(67) $h^2 = (x-x_0)^2 + (y-y_0)^2 = (x-4)^2 + y^2$

$h^2 = (x-4)^2 + (2x-5)$

$h' = 0 \longrightarrow 2(x-4)+2 = 0 \longrightarrow x-4+1=0 \longrightarrow x=3$

$x=3 \longrightarrow h^2 = (3-4)^2 + (1)^2 = 2 \longrightarrow h = \sqrt{2}$

سه نمره 3

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(68) $f'(x) = 2x + \frac{2}{x} \longrightarrow f''(x) = 2 - \frac{2}{x^2} \longrightarrow f''(x)=0 \longrightarrow x=\pm 1$

$m = f'(x_0) = 4$ $\quad (1,1)$ $\quad x=+1$ قابل قبول است

$y - y_0 = m(x-x_0) \longrightarrow y-1 = 4(x-1) \xrightarrow{y=0} x-1 = -\frac{1}{4} \longrightarrow x = \frac{3}{4}$

دو نمره 2

(69) $x = \int_0^y \sqrt{\frac{t}{1-t}}\, dt$ — نرمینه ۲

$$1 = y' \sqrt{\frac{y}{1-y}} \longrightarrow y' = \sqrt{\frac{1-y}{y}} = \sqrt{\frac{1}{y} - 1}$$

$$y'' = \frac{-\frac{y'}{y^2}}{2\sqrt{\frac{1}{y}-1}} = \frac{-y'/y^2}{2y'} = -\frac{1}{2y^2}$$

---

(70) $\lim_{x\to 0} \left(\frac{e^x - 1}{x}\right)^{\frac{1}{x}}$ $\qquad \left(\frac{e^x-1}{x}\right)^{\frac{1}{x}} = A$

$$\ln A = \frac{1}{x}\ln\left(\frac{e^x-1}{x}\right) = \lim_{x\to 0} \frac{\ln\left(\frac{e^x-1}{x}\right)}{x} = \frac{\frac{xe^x-(e^x-1)}{x^2}}{\frac{e^x-1}{x}}$$

$$\lim_{x\to 0} \frac{xe^x - e^x + 1}{x(e^x-1)} = \frac{e^x + xe^x - e^x}{e^x - 1 + x(e^x)} = \frac{e^x + e^x + xe^x - e^x}{e^x + e^x + xe^x} = \frac{e^x}{2e^x} = \frac{1}{2}$$

$\ln A = \frac{1}{2} \rightarrow A = e^{\frac{1}{2}} = \sqrt{e}$

نرمینه ۲

(71) $I = \int_0^{\frac{\pi}{4}} \ln \frac{\cos(\frac{\pi}{4} - x)}{\cos x}\, dx$ $\quad \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{\frac{\pi}{4}} \ln \frac{\cos(\frac{\pi}{4} - (\frac{\pi}{4} - x))}{\cos(\frac{\pi}{4} - x)} = \int_0^{\frac{\pi}{4}} \ln \frac{\cos x}{\cos(\frac{\pi}{4} - x)} = -I \longrightarrow 2I = 0$$

$$I = 0$$

3 نمره

(72) $y = t^4 \longrightarrow x = 2t - \frac{2}{7}t^7$ 1 نمره

$$\begin{cases} x' = 2 - 2t^6 \\ y' = 4t^3 \end{cases} \qquad L = \int \sqrt{x'^2 + y'^2}\, dt$$

$$L = \int \sqrt{(2-2t^6)^2 + 16t^6} = \int \sqrt{4 + 4t^{12} - 8t^6 + 16t^6}$$

$$= \int \sqrt{4t^{12} + 8t^6 + 4}\, dt = 2\int \sqrt{\underbrace{t^{12} + 2t^6 + 1}_{(t^6+1)^2}}\, dt$$

$$= 2\int (t^6 + 1)\, dt = 2\left(\frac{1}{7}t^7 + t\right)_0^1 = 2\left(\frac{1}{7} + 1\right)$$

$$= 2\left(\frac{8}{7}\right) = \frac{16}{7}$$

(73) $z^2 + z \cdot e^{x-3y} - \frac{x^2}{y} + x = 0$

$2z\left(\frac{\partial z}{\partial y}\right) + \frac{\partial z}{\partial y} e^{x-3y} + z\left(-3 e^{x-3y}\right) + \frac{x^2}{y^2} = 0$

$4\left(\frac{\partial z}{\partial y}\right) + \frac{\partial z}{\partial y}(1) + 2(-3) + \frac{9}{1} = 0$

$5\left(\frac{\partial z}{\partial y}\right) + 3 = 0 \longrightarrow \frac{\partial z}{\partial y} = -3/5$

(74) $z = x^2 + xy + y \qquad x + 2y = 12 \longrightarrow x = 12 - 2y$

$z = (12-2y)^2 + (12-2y)y + y$

$\frac{dz}{dy} = 0 \longrightarrow 2(-2)(12-2y) + (12-2y) + (-2y) + 1 = 0$

$-48 + 8y + 12 - 2y - 2y + 1 = 0$

$4y = 35 \longrightarrow y = \frac{35}{4} \Rightarrow x = 12 - 2\times\frac{35}{4} = \frac{-11}{2}$

$z = \frac{11^2}{4} + \left(\frac{-11}{2}\right)\left(\frac{35}{4}\right) + \frac{35}{4} = \frac{2\times121 - 35\times11 + 70}{8}$

(73) $y = e^{-x^2}$ $\quad V = \pi \int x^2 \, dy = \pi \int -\ln y \, dy$ گزینه ۱

$\ln y = -x^2$

$= -\pi \left(y \ln y - y \right) \Big|_1^e = -\pi (0 - 1) = \pi$

$y = e^{-x^2} = \dfrac{1}{e^{x^2}}$

$x = 0 \rightarrow y = 1$

با شرکت در کلاسهای حضوری مهندس فضلی

موفقیت خود در کنکور را از سال آینده تضمین نمایید
