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گروه مهندسی الکترونیک

# نمونه سوالات ریاضیات مهندسی

✓سری های فوریه

2007

الف)

$f(x) = \sin x \qquad\qquad 0 \le x \le \pi$

سری فوریه: $\quad a_0 = \frac{2}{\pi}\int_0^{\pi} \sin x dx = \frac{2}{\pi} \times 2 = \frac{4}{\pi}$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin x \cos 2nx dx = \frac{2}{\pi}\int_0^{\pi} \frac{1}{2}[\sin(1-2n)x + \sin(1+2n)x]dx$$

$$= -\frac{1}{\pi}\left[\frac{\cos(1-2n)x}{1-2n} + \frac{\cos(1+2n)x}{1+2n}\right]_0^{\pi} = -\frac{1}{\pi}\left[\frac{-1}{1-2n} + \frac{-1}{1+2n} - (\frac{1}{1-2n} + \frac{1}{1+2n})\right]$$

$$= -\frac{1}{\pi}\left[\frac{-2}{1-2n} - \frac{2}{1+2n}\right] = \frac{4}{\pi}(\frac{1}{1-4n^2})$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} \sin x \sin 2nx dx = \frac{2}{\pi}\int_0^{\pi} \frac{1}{2}[\cos(2n-1)x - \cos(2n+1)x] = \frac{1}{\Pi}\left[\frac{\sin(2n-1)x}{2n-1} - \frac{\sin(2n+1)x}{2n+1}\right]_0^{\Pi}$$

$$= \frac{1}{\pi}[0] = 0 \Rightarrow \sin x = \frac{8}{\pi} + \sum_{n=1}^{\infty} \frac{4}{\pi}(\frac{1}{1-4n^2}) \cos 2nx$$

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F.C.S

$$\to a_0 = \frac{2}{\pi}\int_0^{\pi} \sin x dx = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin x \cos nx dx = \frac{2}{\pi} = \int_0^{\pi} \frac{1}{2}[\ \sin(1-n)x + \sin(1+n)x\ ] = -\frac{1}{\pi}\left[\frac{\cos(1-n)x}{1-n} + \frac{\cos(1+n)x}{1+n}\right]_0^{\pi}$$

$$= -\frac{1}{\pi}\left[\frac{(-1)^{n+1}}{1-n} + \frac{(-1)^{n+1}}{1+n} - \frac{1}{n-1} - \frac{1}{1+n}\right]$$



چون تابع را زوج در نظر گرفتیم $\to b_n = 0$

$$f(x) = \frac{8}{\pi} + \sum_{n=1}^{\infty} a_n Cosnx$$

F.S.S

$a_n = 0$ $\qquad b_n = \frac{2}{\pi}\int_0^{\pi} \sin x \sin nx = \frac{2}{\pi}\int_0^{\pi} \frac{1}{2}[\cos(1-n)x - \cos(1+n)x]$

$$= \frac{2}{\pi} \times \frac{1}{2}\left[\frac{\sin(1-n)x}{1-n} - \frac{\sin(1+n)x}{1+n}\right]_0^{\pi} = 0 \qquad (n \neq 1)$$

$$n=1 \rightarrow b_1 = \frac{2}{\pi}\int_0^{\pi} \sin^2 x = \frac{2}{\pi}\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\pi} = \frac{2}{\pi}\left(\frac{\pi}{2} - 0\right) = 1$$

$F(x) = 1 \qquad Sin\frac{n\pi}{\pi}x = Sinx$ بدیهی

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$C_n = \frac{1}{2}(a_n - ib_n) = \frac{1}{2}\left(\frac{4}{\pi}\left(\frac{1}{1-4n^2}\right)\right)$ سری مختلط

نمايي:

$$F(x) = \sum_{n=-\infty}^{n=\infty} \frac{2}{\pi}\left(\frac{1}{1-4n^2}\right) \times e^{2nix}$$

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$\beta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) = 0$ دامنه

مرکب:

$$A_n = \sqrt{a_n^2 + b_n^2} = a_n = \frac{4}{\pi}\left(\frac{1}{1-4n^2}\right)$$

$$F(x) = \frac{8}{\pi} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{\frac{\pi}{2}}x - \beta_n\right)$$

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$$\sin x = \frac{8}{\pi} = \sum_{n=1}^{\infty}\left(\frac{4}{\pi\left(1-4n^2\right)}\right)\cos 2nx$$

قضيه دريكله همگرايي

$$x=\frac{\Pi}{2}\rightarrow 1=\frac{8}{\Pi}+\sum_{n=1}^{\infty}\left(\frac{4}{\Pi}\times\frac{1}{1-4n^2}\right)\cos n\Pi$$

$$1-\frac{8}{\pi}=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{1-4n^2}\rightarrow\sum_{n=1}^{\infty}\frac{(-1)^n}{1-4n^2}=\frac{\pi-8}{4}$$

سری مختلط

$$C_n=\frac{a_n-ib_n}{2}$$

غايي

$$a_n=\frac{-a^2}{n^2\pi^2} \qquad b_n=O\Rightarrow C_n=\frac{-a^2}{2n^2\pi^2}$$

$$F(x)=\frac{a^2}{6}+\sum_{\substack{n=-\infty\\ n\neq O}}^{n=\infty}\frac{-a^2}{2n^2\pi^2}e^{\frac{2nin}{a}x}$$

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تابع

$$A_n=\sqrt{\frac{a^2}{\pi^n}+b_n^2}$$

دامنه مرکب:

$$A_n=\sqrt{\frac{a^4}{n^4\pi^4}+O}=\frac{a^2}{n^2\pi^2}$$

$$B_n=Arc\tan\left(\frac{b_n}{a_n}\right)=O\Rightarrow B_n=O\Rightarrow\alpha_n=\frac{\pi}{2}$$

$$=A_0+\sum_{n=1}^{\infty}A_n\left(\cos(\frac{n\pi}{l}x-B_n\right)=\frac{a^2}{6}+\sum_{n=1}^{\infty}\frac{a^2}{n^2\pi^2}\cos\left(\frac{2n\pi}{a}x\right)$$

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$$\frac{1}{l}\int_{-l}^{l}F^2(x)dx=\frac{a_0^2}{2}+\sum_{n=1}^{\infty}\left(a_n^2+b_n^2\right)$$

اتحاد

پارسوال :

$$a_n=\frac{1}{n}[A+B]$$

$$\frac{2}{a}\int_{-a}^{a}\left(ax-x^4\right)^2dx=\int\left(a^4x^2+x^4-2ax^3\right)=\frac{2}{a}\left(a^2\frac{x^3}{3}+\frac{x^5}{5}-\frac{2ax^4}{4}\right]_0^a$$

$$=\frac{2}{a}\left(\frac{a^5}{3}+\frac{a^5}{5}-\frac{a^5}{2}\right)=2a^4\left(\frac{1}{3}+\frac{1}{5}-\frac{1}{2}\right)=\frac{2a^4}{30}=\frac{a^4}{15}$$

$$\frac{a^4}{15}=\frac{a^4}{18}+\sum_{n=1}^{\infty}\frac{a^4}{n^4\pi^4}\Rightarrow\frac{a^4}{15}-\frac{a^4}{18}=\frac{a^4}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4}\Rightarrow\sum_{n=1}^{\infty}=\left(\frac{a^4}{15}-\frac{a^4}{18}\right)\times\frac{\pi^4}{a^4}$$

$$A=\left[\left(ax^2-x^2\right)\left(\frac{a}{n\pi}Sin\frac{n\pi}{a}x\right)+(a\quad x)\left(-\right.\right.$$

$$A=\begin{cases}\dfrac{-2a^3}{n^2\pi^2} & \text{even}\\ 0 & \text{odd}\end{cases}$$

$$B=\left[\left(ax+x^2\right)\frac{a}{n\pi}Sin\frac{n\pi}{a}x\right)+(a+2x)\frac{a^2}{n^2\pi^2}Cos\frac{n\pi}{a}x-\frac{2a^3}{n^3\pi^3}Sin\frac{n\pi}{a}x\Big|_{-a}^{0}$$

$$B=\begin{cases}\dfrac{-2a^3}{n^2\pi^2} & \text{even}\\ 0 & \text{odd}\end{cases}$$



$$b_n=\frac{1}{a}\left[\int_0^a(ax-x^2)Sin\frac{n\pi}{a}x+\int_{-a}^{0}(ax+x^2)Sin\frac{n\pi}{a}x\right]\Rightarrow b_n=\frac{1}{a}[A+B]$$



$$A=\left[\left(-ax+x^2\right)\frac{a}{n\pi}Cos\frac{n\pi}{a}x+(a-2x)\frac{a^2}{n^2\pi^2}Sin\frac{n\pi}{a}x-\frac{2a^3}{n^3\pi^3}Cos\frac{n\pi}{a}\right]_0^a$$

$$A=\frac{-2a^3}{n^3\pi^3}(-1)^n+\frac{2a^3}{n^3\pi^3}=\begin{cases}0 & \text{even}\\ \ & \end{cases}$$

$$\frac{4a^3}{n^3\pi^3} \qquad \text{odd}$$

$$B=\left[\left(ax+x^2\right)\left(\frac{-a}{n\pi}Cos\frac{n\pi}{a}x\right)+\left(a+2x\right)\left(\frac{a^2}{n^2\pi^2}Sin\frac{n\pi}{a}x\right)+\frac{2a^3}{n^3\pi^3}Cos\frac{n\pi}{a}x\right]_{-a}^{0}$$

$$B=\frac{2a^3}{n^3\pi^3}+\frac{2a^3}{n^3\pi^3}(-1)^n=$$

→ $\frac{4a^3}{n^3\pi^3}$ even

→ 0 odd

$$b_n=\frac{1}{a}\times\frac{4a^3}{n^3\pi^3}=\frac{4a^2}{n^3\pi^3}$$

$$F(x)=\sum_{n=1}^{\infty}\frac{4a^2}{n^3\pi^3}\sin\frac{n\pi}{a}x$$ سری سینوسی

فوریه

$$a_0=\frac{2}{a}\int_0^a\left(ax-x^2\right)dx=\frac{2}{a}\left[\frac{ax^2}{2}-\frac{x^3}{3}\right]\Bigg|_0^a=\frac{2}{a}\left[\frac{a^3}{2}-\frac{a^3}{3}\right]=\frac{2}{a}\times\frac{a^3}{6}=\frac{a^2}{3}$$

$$a_n=\frac{2}{a}\int_0^a\left(ax-x^2\right)\cos\frac{2n\pi}{a}x\,dx=\left(ax-x^2\right)\left(\frac{a}{2n\pi}\sin\frac{2n\pi}{a}x\right)+\left(a-2x\right)\left(\frac{a^2}{4n^2\pi^2}\cos\frac{2n\pi}{a}x\right)$$

$$+\frac{2a^3}{8n^3\pi^3}\sin\frac{2n\pi}{a}x\Bigg|_0^a\Rightarrow a_n=\frac{-a^2}{n^2\pi^2}$$

$$b_n=\frac{2}{a}\int_0^a\left(ax-x^2\right)\sin\frac{2n\pi}{a}x\,dx=$$

$$\left(ax-x^2\right)\times\left(\frac{-a}{2n\pi}\cos\frac{2n\pi}{a}x\right)+\left(a-2x\right)\times\frac{a^3}{4n\pi}\sin\frac{2n\pi}{a}x-\frac{a^6}{4n^3\pi^3}\cos\frac{2n\pi}{ax}\Bigg|_0^a$$

$$b_n=0$$

$$ax-x^2=\frac{a^2}{6}+\sum_{n=1}^{\infty}\left(\frac{-a^2}{n^2\pi^2}\right)\cos\frac{2n\pi}{a}x$$

$$F(x)=\begin{cases} ax-x^2 & 0\le x\le a \\ ax+x^2 & -a\le x\le a \end{cases}$$

$$a_0=\frac{1}{a}\left[\int_0^a (ax-x^2)dx+\int_{-a}^0 (ax+x^2)dx\right]$$

$$a_0=\frac{1}{a}\left[\left[\frac{ax^2}{2}-\frac{x^3}{3}\right]\Bigg|_0^a+\left[\frac{ax^2}{2}+\frac{ax^3}{3}\right]\Bigg|_{-a}^0\right]$$

$$=\frac{1}{a}\left[\frac{a^3}{6}-\frac{a^3}{6}\right]=0$$

$$a_n=\frac{1}{a}\left[\int_0^a (ax-x^2)Cos\frac{n\pi}{a}xdx+\int_{-a}^0 (ax+x^2Cos\frac{n\pi}{a}xdx\right]=$$

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الف)

3) $f(x)=1+|x| \qquad -l<x<l$

$b_n=0\Rightarrow$ چون تابع زوج است.

$$a_n=\frac{1}{l}\int_{-l}^{l}(1+|x|)Cos\frac{n\pi}{l}xdx$$

$$=\frac{1}{l}\left[\int_{-l}^{l}Cos\frac{n\pi}{l}xdx+\int_{-l}^{l}|x|Cos\frac{n\pi}{l}xdx\right]$$



$$\mathrm{I}=\frac{l}{n\pi}Sin\frac{n\pi}{l}\Big]_{-l}^{l}=0$$



I II

II:
$$\int_{-l}^{l}|x|Cos\frac{n\pi}{l}xdx=\int_{-l}^{0}-xCos\frac{n\pi}{l}xdx+\int_0^l xCos\frac{n\pi}{l}xdx$$

$$=2\left[\frac{lx}{n\pi}Sin\frac{n\pi}{l}x+\frac{l^2}{n^2\pi^2}Cos\frac{n\pi}{l}x\right]_0^l=\left[\frac{l^2}{n^2\pi^2}((-1^n)-1)\right]$$

$$f(x)=\frac{\frac{1}{l}\int_{-l}^{l}(1+|x|)dx}{2}+\sum_{n=1}^{\infty}2\left[\frac{l^2}{n^2\pi^2}\left((-1)^n-1\right)\right]Cos\frac{n\pi}{l}x$$

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F.S.S $$F(x)=\begin{cases}x+1 & 0\le x\le 1\\ -x+3 & 1<x\le 2\end{cases}$$

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F.C.S → سری کسینوسی این تابع همان سری فوریه اش می شود

مختلط نمائی فوریه:

$$C_n=\frac{1}{2}(a_n-ib_n)=\frac{1}{2}\left[2\left[\frac{l^2}{n^2\pi^2}((-1)n-1)\right]\right]$$

$$F(x)=\sum_{n=-\infty}^{n=\infty}C_n e^{\frac{ni\Pi}{l}x}$$

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دامنه مرکب فوریه:

$$\beta_n=\tan^{-1}\left(\frac{b_n}{a_n}\right)=0$$

$$A_n=\sqrt{a_n^2+b_n^2}=a_n$$

$$a_0=\frac{1}{l}\int_{-l}^{l}(1+|x|)dx=\frac{1}{l}\left[\int_{-l}^{0}(1-x)dx+\int_{0}^{l}(1+x)dx\right]=$$

$$=\frac{1}{l}\left[\left[x-\frac{x}{2}\right]_{-l}^{0}+\left[x+\frac{x}{2}\right]_{0}^{l}\right]=\frac{2l+l^2}{l}=2+l$$

$$F(x)=\frac{2+l}{2}+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{l}x-\beta_n\right)$$



ب) 3 قضیه دریکله

$$1+|x|=\frac{2+l}{2}+\sum_{n=1}^{\infty}\left[\frac{l^2}{n^2\pi^2}\left((-1)^n-1\right)\right]\cos\frac{n\pi}{l}x$$

$$x=1\rightarrow 2=1+\frac{l}{2}+\sum_{n=1}^{\infty}\left[2\frac{l^2}{n^2\pi^2}\left((-1)^n-1\right)\right]\cos n\pi$$

$$1-\frac{l}{2}=\frac{2l^2}{n^2\pi^2}\sum_{n=1}^{\infty}\left((-1)^n-1\right)(-1)^n$$

$$\rightarrow\sum_{n=1}^{\infty}(-1)^{2n}-(-1)^n=\frac{1-\frac{l}{2}}{\frac{2l^2}{n^2\pi^2}}$$

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الف) 4

سری فوریه $F(x)=\cos\alpha x \qquad 0<x<\pi$ :

$$a_n=\frac{2}{\pi}\int_0^{\pi}\cos\alpha x\cos 2nx\,dx=\frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\cos(2n-\alpha)x+\cos(2n+\alpha)x\right]$$

$$=\frac{1}{\pi}\left[\frac{\sin(2n-\alpha)x}{(2n-\alpha)}+\frac{\sin(2n-\alpha)x}{(2n+\alpha)}\right]_0^{\pi}=\frac{1}{\pi}\left[\frac{\sin(2n-\alpha)\pi}{(2n-\alpha)}+\frac{\sin(2n+\alpha)\pi}{(2n+\alpha)}\right]=I$$

لم: $\sin\left(2n\pm\alpha\right)\pi=\sin 2n\pi\cos\alpha\pi\pm\cos 2n\pi\sin\alpha\pi=\pm\sin\alpha\pi$

$$I=\frac{1}{\pi}\left[\frac{\sin\alpha\pi}{2n+\alpha}+\frac{-\sin\alpha\pi}{(2n-\alpha)}\right]=\frac{\sin\alpha\pi}{\pi}\left[\frac{-2\alpha}{(4n-\alpha)}\right]=a_n$$

$$b_n=\frac{2}{\pi}\int_0^{\pi}\cos\alpha x\sin 2nx\,dx=\frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\sin(2n-\alpha)x+\sin(2n+\alpha)x\right]$$

$$=\frac{-1}{\pi}\left[\frac{\cos(2n-\alpha)x}{(2n-\alpha)}+\frac{\cos(2n+\alpha)x}{(2n+\alpha)}\right]_0^{\pi}=-\frac{1}{\pi}\left[\frac{\cos(2n-\alpha)\pi}{(2n-\alpha)}+\frac{\cos(2n+\alpha)\pi}{(2n+\alpha)}-\frac{4n}{4n^2-\alpha^2}\right]_0^{\pi}$$



لم:

$$\cos(2n \mp \alpha)\pi = \cos 2n\pi \cos \alpha\pi \pm \sin 2n\pi \sin \alpha\pi = \cos \alpha\pi$$

$$b_n = -\frac{1}{\pi}\left[\frac{\cos\alpha\pi}{2n-\alpha} + \frac{\cos\alpha\pi}{2n+\alpha} - \frac{4n}{4n^2-\alpha^2}\right]$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} \cos\alpha x dx = \frac{2}{\pi}\left[\frac{1}{\alpha}\sin\alpha x\right]_0^{\pi} = \frac{2}{\alpha\pi}(\sin\alpha\pi)$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos^2 nx + b_n \sin 2nx\right)$$

F.C.S $\qquad b_n = 0 \qquad T = 2\pi \rightarrow l = \pi$

سری کسینوس فوریه

$$a_n = \frac{2}{\pi}\int_0^{\pi} \cos\alpha x \cos nx dx = \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\cos(n-\alpha)x + \cos(n+\alpha)x\right]$$

$$= \frac{1}{\pi}\left[\frac{\sin(n-\alpha)x}{n-\alpha} + \frac{\sin(n+\alpha)x}{n+\alpha}\right]_0^{\pi} = \frac{1}{\pi}\left[\frac{\sin(n-\alpha)\pi}{n-\alpha} + \frac{\sin(n+\alpha)\pi}{n+\alpha}\right]$$

لم: $\sin(n \pm \alpha)\pi = \sin n\pi \cos\alpha\pi \pm \cos n\pi \sin\alpha\pi = \pm (-1)^n \sin\alpha\pi$

$$a = \frac{1}{\pi}(-1)^n \sin\alpha\pi\left[\frac{1}{n+\alpha} + \frac{1}{n+\alpha}\right] = \frac{2\alpha\sin\alpha\pi}{\pi(\alpha^2-n^2)}(-1)^n$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a\cos nx$$

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F.S.S $\qquad b_n = \frac{2}{\pi}\int_0^{\pi}\cos\alpha x \sin nx dx$

$a_n = 0 \qquad b_n = \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\sin(n-\alpha)\pi + \sin(n+\alpha)\pi\right]^{dx}$

$$b_n = \frac{-1}{\pi}\left[\frac{\cos(n-\alpha)\pi}{n-\alpha} + \frac{\cos(n+\alpha)\pi}{n+\alpha} - \frac{1}{n-\alpha} - \frac{1}{n+\alpha}\right]$$

لم: $\cos(n \mp \alpha)\pi = \cos n\pi \cos\alpha\pi \pm \sin n\pi \sin\alpha\pi = (-1)^n \cos\alpha\pi$

$$b_n = -\frac{1}{\pi}\left[\frac{(-1)^n\cos\alpha\pi}{n-\alpha} + \frac{(-1)^n\cos\alpha\pi}{n+\alpha} - \frac{1}{n-\alpha} - \frac{1}{n+\alpha}\right]$$

$$F(x) = \frac{a_\circ}{2} + \sum_{n=1}^{\infty} b\sin nx$$